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ASYMPTOTIC EXPANSIONS OF SOLUTIONS OF THE FIRST INITIAL BOUNDARY-VALUE PROBLEM FOR SCHRÖDINGER SYSTEMS IN DOMAINS WITH CONICAL POINTS. II*

АСИМПТОТИЧН РОЗКЛАДИ РОЗВ'ЯЗКІВ ПЕРШОЇ ПОЧАТКОВОЇ КРАЙОВОЇ ЗАДАЧІ ДЛЯ СИСТЕМ ШРЕДІНГЕРА В ОБЛАСТЯХ З КОНІЧНИМИ ТОЧКАМИ. II

This paper is concerned with asymptotic expansions of solutions of the first initial boundary-value problem for strongly Schrödinger systems near a conical point of the domain boundary.

Розглянуто асимптотичні розклади розв'язків першої початкової крайової задачі для сильно шредінгерових систем біля конічної точки межі області.

1. Introduction and notations. At the present there exists a comprehensive theory of boundary-value problems for elliptic, parabolic, and hyperbolic equations and systems with a smooth boundary. One of the central results of this theory consists in the fact that if the coefficients of the equation and of the boundary operators, its right-hand side, and the boundary of the domain are sufficiently smooth, then the solution itself of the problem is correspondingly smooth. (In the parabolic and hyperbolic cases, the initial and boundary conditions must also satisfy the so-called compatibility condition; see, e.g., [1, 2]).

However, many important applied problems reduce to the study of boundary-value problems for partial differential equations in non-smooth domains. Such questions have been discussed extensively in the literature since the appearance of the fundamental work [3] of Kondartiev in 1967. By now the theory of boundary-value problems for elliptic equations in non-smooth domains has been worked out in much detail, with a large literature on it. We refer the survey paper [4] and the monographs [5, 6] for the results. Parallel with this theory, the boundary-value problems for non-stationnary equations and systems have been studied by many athors, such as Melinikov [7], Ngok [8], Eskin [9], Kokotov and Plamenevskii [10, 11], Matyukevich and Plamenevskii [12], ... In these works, they used results and methods of elliptic boundary-value problems in nonsmooth domains to prove the assertions on the unique solvability, on the smoothness and the asymptotic expansions of solutions near the singularities on the boundary.

Boundary-value problems for Schrödinger equations and Schrödinger systems in a finite cylinder $\Omega_T = \Omega \times (0, T)$ have been studied by many authors (see, e.g., [1, 2, 13, 14]). In this paper, we continue the investigation presented in [15 – 18], in which we considered the first initial boundary-value problem for strongly Schrödinger systems in an infinite cylinder $\Omega_\infty = \Omega \times (0, \infty)$, where Ω is a bounded domain with conical points. The existence, uniqueness and smoothness of generalized solutions to the problem were

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given in [15, 16, 18]. The aim of [17] and this paper is to derive the asymptotic expansion of the generalized solution of this problem in a neighbourhood of the singular point.

Let Ω be a bounded domain in $\mathbb{R}^n$. Its boundary $\partial\Omega$ is assumed to be an infinitely differentiable surface everywhere except the coordinate origin, in a neighbourhood of which Ω coincides with the cone $K = \{x\colon x/|x| \in G\}$, where G is a smooth domain on the unit sphere S^{n-1}. We begin by recalling some notations and functional spaces which will be frequenly used in this paper:

$\Omega_T = \Omega \times (0,T)$, $S_T = \partial\Omega \times (0,T)$, $\Omega_\infty = \Omega \times (0,\infty)$, $S_\infty = \partial\Omega \times (0,\infty)$, $x = (x_1,\ldots,x_n) \in \Omega$, $u(x,t) = (u_1(x,t),\ldots,u_s(x,t))$ is a vector complex function,

$$|D^\alpha u|^2 = \sum_{i=1}^{s} |D^\alpha u_i|^2, \qquad u_{t^j} = \left(\frac{\partial^j u_1}{\partial t^j},\ldots,\frac{\partial^j u_s}{\partial t^j}\right),$$

$$|u_{t^j}|^2 = \sum_{i=1}^{s} \left|\frac{\partial^j u_i}{\partial t^j}\right|^2, \qquad dx = dx_1\ldots dx_n, \quad r = |x| = \sqrt{x_1^2+\ldots+x_n^2};$$

$H^l_\beta(\Omega)$ — the space of all functions $u(x) = (u_1(x),\ldots,u_s(x))$ which have generalized derivatives $D^\alpha u_i$, $|\alpha| \leqslant l$, $1 \leqslant i \leqslant s$, satisfying

$$\|u\|^2_{H^l_\beta(\Omega)} = \sum_{|\alpha|=0}^{l} \int_\Omega r^{2(\beta+|\alpha|-l)} |D^\alpha u|^2 dx < +\infty;$$

$H^{l,k}(e^{-\gamma t},\Omega_\infty)$ — the space of all functions $u(x,t)$ which have generalized derivatives $D^\alpha u_i$, $\dfrac{\partial^j u_i}{\partial t^j}$, $|\alpha| \leqslant l$, $1 \leqslant j \leqslant k$, $1 \leqslant i \leqslant s$, satisfying

$$\|u\|^2_{H^{l,k}(e^{-\gamma t},\Omega_\infty)} = \int_{\Omega_\infty} \left(\sum_{|\alpha|=0}^{l} |D^\alpha u|^2 + \sum_{j=1}^{k} |u_{t^j}|^2\right) e^{-2\gamma t} dxdt < +\infty;$$

in particular

$$\|u\|^2_{H^{l,0}(e^{-\gamma t},\Omega_\infty)} = \sum_{|\alpha|=0}^{l} \int_{\Omega_\infty} |D^\alpha u|^2 e^{-2\gamma t} dxdt;$$

$\overset{\circ}{H}{}^{l,k}(e^{-\gamma t},\Omega_\infty)$ — the closure in $H^{l,k}(e^{-\gamma t},\Omega_\infty)$ of the set of all infinitely differentiable in Ω_∞ functions which belong to $H^{l,k}(e^{-\gamma t},\Omega_\infty)$ and vanish near S_∞;

$H^{l,k}_\beta(e^{-\gamma t},\Omega_\infty)$ — the space of all functions $u(x,t)$ which have generalized derivatives $D^\alpha u_i$, $\dfrac{\partial^j u_i}{\partial t^j}$, $|\alpha| \leqslant l$, $1 \leqslant j \leqslant k$, $1 \leqslant i \leqslant s$, satisfying

$$\|u\|^2_{H^{l,k}_\beta(e^{-\gamma t},\Omega_\infty)} = \int_{\Omega_\infty} \left(\sum_{|\alpha|=0}^{l} r^{2(\beta+|\alpha|-l)} |D^\alpha u|^2 + \sum_{j=1}^{k} |u_{t^j}|^2\right) e^{-2\gamma t} dxdt < +\infty;$$

$H^l_\beta(e^{-\gamma t},\Omega_\infty)$ — the space of all functions $u(x,t)$ which have generalized derivatives $D^\alpha (u_i)_{t^j}$, $|\alpha| + j \leqslant l$, $1 \leqslant i \leqslant s$, satisfying

$$\|u\|^2_{H^l_\beta(e^{-\gamma t},\Omega_\infty)} = \sum_{|\alpha|+j=0}^{l} \int_{\Omega_\infty} r^{2(\beta+|\alpha|+j-l)} |D^\alpha u_{t^j}|^2 e^{-2\gamma t} dxdt < +\infty;$$

Let X be a Banach space. Denote by $L^\infty(0,\infty;X)$ the space consisting of all measurable functions $u\colon (0,\infty)\longrightarrow X$, $t\longmapsto u(x,t)$ satisfying

$$\|u\|_{L^\infty(0,\infty;X)} = \operatorname*{ess\,sup}_{t>0}\big\|u(x,t)\big\|_X < +\infty.$$

Consider the differential operator of order $2m$

$$L(x,t,D) = \sum_{|p|,|q|=0}^{m} D^p\big(a_{pq}(x,t)D^q\big),$$

where a_{pq} are $s\times s$-matrices of measurable bounded in $\overline{\Omega}_\infty$ complex functions, $a_{pq} = (-1)^{|p|+|q|}a^*_{qp}$. Suppose that a_{pq} are continuous in $x\in\overline{\Omega}$ uniformly with respect to $t\in[0,\infty)$ if $|p|=|q|=m$, and for each $t\in[0,\infty)$ the operator $L(x,t,D)$ is uniformly elliptic in $\overline{\Omega}$ with ellipticity constant a_0 independent of time t, i.e., we have

$$\sum_{|p|=|q|=m} a_{pq}(x,t)\xi^p\xi^q\eta\overline{\eta} \ge a_0|\xi|^{2m}|\eta|^2,$$

for all $\xi\in\mathbb{R}^n\setminus\{0\}$, $\eta\in\mathbb{C}^s\setminus\{0\}$ and $(x,t)\in\overline{\Omega}_\infty$.

In this paper we study the following problem: Find a function $u(x,t)$ such that

$$(-1)^{m-1}iL(x,t,D)u - u_t = f(x,t) \quad \text{in} \quad \Omega_\infty, \tag{1.1}$$

$$u|_{t=0} = 0, \tag{1.2}$$

$$\frac{\partial^j u}{\partial\nu^j}\Big|_{S_\infty} = 0, \quad j=0,\ldots,m-1, \tag{1.3}$$

where ν is the outer unit normal to S_∞.

A function $u(x,t)$ is called a generalized solution of the problem (1.1)–(1.3) *in the space $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty)$ if and only if $u(x,t)$ belongs to $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty)$ and for each $T>0$ the following equality holds*

$$(-1)^{m-1}i\sum_{|p|,|q|=0}^{m}(-1)^{|p|}\int\limits_{\Omega_T} a_{pq}D^q u\overline{D^p\eta}\,dxdt + \int\limits_{\Omega_T} u\overline{\eta}_t\,dxdt = \int\limits_{\Omega_T} f\overline{\eta}\,dxdt, \tag{1.4}$$

for all test function $\eta\in\overset{\circ}{H}{}^{m,1}(\Omega_T)$, $\eta(x,T)=0$.

Putting

$$B(u,u)(t) = \sum_{|p|,|q|=0}^{m}(-1)^{|p|}\int\limits_{\Omega} a_{pq}D^q u\overline{D^p u}\,dx, \quad u(x,t)\in\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty).$$

For a.e. $t\in[0,\infty)$, the function $x\mapsto u(x,t)$ belongs to $\overset{\circ}{H}{}^{m}(\Omega)$. On the other hand, since the principal coefficients a_{pq} are continuous in $x\in\overline{\Omega}$ uniformly with respect to $t\in[0,\infty)$ and the ellipticity constant a_0 independent of t, repeating the proof of Garding's inequality [19, p. 44] we have the following lemma.

Lemma 1.1. *There exist two constants μ_0 and λ_0 $(\mu_0>0, \lambda_0\ge 0)$ such that*

$$(-1)^m B(u,u)(t) \ge \mu_0\big\|u(x,t)\big\|^2_{H^m(\Omega)} - \lambda_0\big\|u(x,t)\big\|^2_{L_2(\Omega)} \tag{1.5}$$

for all $u(x,t)\in\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty)$.

Therefore, using the transformation $u = e^{i\lambda_0 t}v$ if necessary, we can assume that the operator $L(x,t,D)$ satisfies

$$(-1)^m B(u,u)(t) \geq \mu_0 \|u\|^2_{H^m(\Omega)} \tag{1.6}$$

for all $u(x,t) \in \overset{\circ}{H}{}^{m,0}(e^{-\gamma t}, \Omega_\infty)$. This inequality is a basic tool for proving the existence and uniqueness of solutions of the problem.

2. Smoothness of generalized solutions. In this section we summarize the known results on the smoothness of generalized solutions of the problem (1.1)–(1.3).

Denote by m^* the number of multiindexes which have order not exceeding m, μ_0 is the constant in (1.6). The following theorem was proved in [18].

Theorem 2.1. *Let*

i) $\sup\left\{\left|\dfrac{\partial a_{pq}}{\partial t}\right| : (x,t) \in \overline{\Omega}_\infty\ 0 \leqslant |p|,\ |q| \leqslant m\right\} = \mu < +\infty;\ \left|\dfrac{\partial^k a_{pq}}{\partial t^k}\right| \leqslant \mu_1$, $\mu_1 = \text{const} > 0$, *for* $2 \leqslant k \leqslant h+1$;

ii) $f_{t^k} \in L^\infty(0,\infty; L_2(\Omega))$, *for* $k \leqslant h+1$;

iii) $f_{t^k}(x,0) = 0$, *for* $k \leqslant h$.

Then for every $\gamma > \gamma_0 = \dfrac{m^*\mu}{2\mu_0}$, *the problem* (1.1)–(1.3) *has exactly one generalized solution* $u(x,t)$ *in the space* $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t}, \Omega_\infty)$. *Moreover,* $u(x,t)$ *has derivatives with respect to* t *up to order* h *belonging to* $\overset{\circ}{H}{}^{m,0}(e^{-(2h+1)\gamma t}, \Omega_\infty)$ *and the following estimate holds*:

$$\|u_{t^h}\|^2_{H^{m,0}(e^{-(2h+1)\gamma t}, \Omega_\infty)} \leqslant C \sum_{k=0}^{h+1} \|f_{t^k}\|^2_{L^\infty(0,\infty;L_2(\Omega))},$$

where C *is a positive constant independent of* u *and* f.

From now forward, for the sake of brevity we will write γ_h instead of $(2h{+}1)\gamma$, $h = 1, 2, \ldots$.

In order to study the smoothness with respect to (x,t) and to establish asymptotic formulas of solutions of the problem (1.1)–(1.3), we assume that coefficients $a_{pq}(x,t)$ of the operator $L(x,t,D)$ are infinitely differentiable in $\overline{\Omega}_\infty$. Moreover, we also assume that $a_{pq}(x,t)$ and its all derivatives are bounded in $\overline{\Omega}_\infty$.

First, we recall two basic lemmas.

Lemma 2.1 [16]. *Let* $f, f_t, f_{tt} \in L^\infty(0,\infty; L_2(K))$ *and* $f(x,0) = f_t(x,0) = 0$. *If* $u(x,t) \in \overset{\circ}{H}{}^{m,0}(e^{-\gamma t}, \Omega_\infty)$ *is a generalized solution of the problem* (1.1)–(1.3) *in the space* $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t}, \Omega_\infty)$ *such that* $u \equiv 0$ *whenever* $|x| > R = \text{const}$, *then* $u \in H^{2m,0}_m(e^{-\gamma_1 t}, K_\infty)$ *and the following estimate holds*:

$$\|u\|^2_{H^{2m,0}_m(e^{-\gamma_1 t}, K_\infty)} \leqslant$$

$$\leqslant C\left[\|f\|^2_{L^\infty(0,\infty;L_2(K))} + \|f_t\|^2_{L^\infty(0,\infty;L_2(K))} + \|f_{tt}\|^2_{L^\infty(0,\infty;L_2(K))}\right],$$

where $C = \text{const}$.

Denote by $L_0(0,t,D)$ the principal part of the operator $L(x,t,D)$ at origin 0. We consider the Dirichlet problem for the system

$$(-1)^{m-1} L_0(0,t,D)u = F(x,t), \quad x \in K. \tag{2.1}$$

Lemma 2.2 [16]. *Let $u(x,t)$ be a generalized solution of the Dirichlet problem for the system* (2.1) *for a.e.* $t \in [0,\infty)$ *such that* $u \equiv 0$ *whenever* $|x| > R = \text{const}$, *and* $u(x,t) \in H^{2m+l-1,0}_{\beta-1}(e^{-\gamma t}, K_\infty)$. *Let* $F \in H^{l,0}_{\beta}(e^{-\gamma t}, K_\infty)$. *Then* $u(x,t) \in \in H^{2m+l,0}_{\beta}(e^{-\gamma t}, K_\infty)$ *and*

$$\|u\|^2_{H^{2m+l,0}_{\beta}(e^{-\gamma t},K_\infty)} \leqslant C\left[\|F\|^2_{H^{l,0}_{\beta}(e^{-\gamma t},K_\infty)} + \|u\|^2_{H^{2m+l-1,0}_{\beta-1}(e^{-\gamma t},K_\infty)}\right],$$

where $C = \text{const}$.

Let ω be a local coordinate system on S^{n-1}. The principal part of the operator $L(x,t,D)$ at origin 0 can be written in the form

$$L_0(0,t,D) = r^{-2m}Q(\omega,t,rD_r,D_\omega), \qquad D_r = \frac{i\partial}{\partial r},$$

where Q is a linear operator with smooth coefficients. From now forward the following spectral problem will play an important role

$$Q(\omega,t,\lambda,D_\omega)v(\omega) = 0, \quad \omega \in G, \tag{2.2}$$

$$D^j_\omega v(\omega) = 0, \qquad \omega \in \partial G, \quad j = 0,\ldots,m-1. \tag{2.3}$$

It is well known [5, p. 146] that for every $t \in [0,\infty)$ its spectrum is discrete.

Theorem 2.2 [16]. *Let $u(x,t)$ be a generalized solution of the problem* (1.1) – (1.3) *in the space* $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty)$ *and let* $f_{t^k} \in L^\infty(0,\infty; H^l_0(\Omega))$ *for* $k \leqslant 2m+l+1$, $f_{t^k}(x,0) = 0$ *for* $k \leqslant 2m+l$. *In addition supppose that the strip*

$$m - \frac{n}{2} \leqslant \operatorname{Im}\lambda \leqslant 2m + l - \frac{n}{2}$$

does not contain points of spectrum of the problem (2.2), (2.3) *for every* $t \in [0,\infty)$. *Then* $u(x,t) \in H^{2m+l}_0(e^{-\gamma_{2m+l}t},\Omega_\infty)$ *and the following estimate holds*:

$$\|u\|^2_{H^{2m+l}_0(e^{-\gamma_{2m+l}t},\Omega_\infty)} \leqslant C\sum_{k=0}^{2m+l+1}\|f_{t^k}\|^2_{L^\infty(0,\infty;H^l_0(\Omega))},$$

where $C = \text{const}$.

3. Asymptotic expansions of generalized solutions. In this section we will derive asymptotic expansions of generalized solutions of the problem (1.1) – (1.3) in a neighbourhood of the conical point, in the case that the condition imposed on the spectrum of the problem (2.2), (2.3) in Theorem 2.2 is not satisfied. The following result was obtained in [17].

Theorem 3.1. *Let $u(x,t)$ be a generalized solution of the problem* (1.1) – (1.3) *in the spaces* $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty)$, *and let* $f_{t^k} \in L^\infty(0,\infty;H^l_0(\Omega))$ *for* $k \leqslant l+2m+1$, $f_{t^k}(x,0) = 0$ *for* $k \leqslant l+2m$. *Assume that in the strip* $m - \dfrac{n}{2} \leqslant \operatorname{Im}\lambda \leqslant 2m+l-\dfrac{n}{2}$, *there exists only one simple eigenvalue $\lambda(t)$ of the problem* (2.2), (2.3) *such that*

$$2m + l - 1 - \frac{n}{2} < \operatorname{Im}\lambda(t) < 2m + l - \frac{n}{2}.$$

Then the following representation holds:

$$u(x,t) = c(x,t)r^{-i\lambda(t)} + u_1(x,t),$$

where $c(x,t) \in V^{2m+l}_{\operatorname{Im}\lambda(t)}(e^{-\gamma_{2m+l}t},\,\Omega_\infty)$, $u_1 \in H^{2m+l}_0(e^{-\gamma_{2m+l}t},\Omega_\infty)$.

Attention is now turned to the case that the strip $m - \dfrac{n}{2} \leqslant \operatorname{Im}\lambda \leqslant 2m + l - \dfrac{n}{2}$ contains finite simple eigenvalues $\lambda_1(t), \ldots, \lambda_{N_0}(t)$ of the spectral problem (2.2), (2.3).

Consider in K the Dirichlet problem for the system

$$L_0(0,t,D)u = r^{-i\lambda_0(t)-2m} \sum_{s=0}^{M} \ln^s r f_s(\omega, t), \tag{3.1}$$

where ω is a local coordinate system on S^{n-1}.

Lemma 3.1 [4, p. 17]. *Let $f_s(\omega,t)$, $s = 0, \ldots, M$, be infinitely differentiable functions of ω. Then there exists a solution of the Dirichlet problem for the system* (3.1) *having the form*

$$u(x,t) = r^{-i\lambda_0(t)} \sum_{s=0}^{M+\mu} \ln^s r \tilde{f}_s(\omega, t),$$

where $\tilde{f}_s$, $s = 0, \ldots, M + \mu$, are the infinitely differentiable functions of ω, $\mu = 1$ if λ_0 is a simple eigenvalue of the problem (2.2), (2.3) *and $\mu = 0$ if λ_0 is not an eigenvalue of this problem.*

From now forward, we denote

$$L_{2,\mathrm{loc}}[0,\infty) = \Big\{c(t)\colon c(t) \in L_2[0,T] \ \text{ for all } \ T > 0\Big\}.$$

Lemma 3.2. *Let $u(x,t)$ be a generalized solution of the Dirichlet problem for the system* (2.1) *for a.e. $t \in [0,\infty)$ such that $u \equiv 0$ whenever $|x| > R = \mathrm{const}$, and let $u_{t^k} \in H_{\beta}^{2m+l,0}(e^{-\gamma_k t}, K_\infty)$, $F_{t^k} \in H_{\beta'}^{l,0}(e^{-\gamma_k t}, K_\infty)$ for $k \leqslant h$, $\beta' < \beta \leqslant m + l$. Assume that the straight lines*

$$\operatorname{Im}\lambda = -\beta + 2m + l - \frac{n}{2} \quad \textit{and} \quad \operatorname{Im}\lambda = -\beta' + 2m + l - \frac{n}{2}$$

do not contain points of spectrum of the problem (2.2), (2.3) *for every $t \in [0,\infty)$, and in the strip*

$$-\beta + 2m + l - \frac{n}{2} < \operatorname{Im}\lambda < -\beta' + 2m + l - \frac{n}{2}$$

there exists only one simple eigenvalue $\lambda(t)$ of the problem (2.2), (2.3). *Then the following representation holds*:

$$u(x,t) = c(t) r^{-i\lambda(t)} \phi(\omega,t) + u_1(x,t), \tag{3.2}$$

where ϕ is an infinitely differentiable function of (ω,t) which does not depend on the solution, $c_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$ and $(u_1)_{t^k} \in H_{\beta'}^{2m+l,0}(e^{-\gamma_k t}, K_\infty)$ for $k \leqslant h$.

Proof. From Theorem 3.2 in [20], it follows that

$$u(x,t) = c(t) r^{-i\lambda(t)} \phi(\omega,t) + u_1(x,t), \tag{3.3}$$

where $\phi(\omega,t)$ is the eigenfunction of the problem (2.2), (2.3) which correspond to the eigenvalue $\lambda(t)$, $u_1 \in H_{\beta'}^{2m+l,0}(e^{-\gamma t}, K_\infty)$, and

$$c(t) = i \int\limits_K F(x,t) r^{-i\overline{\lambda(t)}+2m-n} \psi(x,t)dx,$$

where ψ is the eigenfunction of the problem conjugating to the problem (2.2), (2.3) and which corresponds to the eigenvalue $\overline{\lambda(t)}$. Since $\operatorname{Im} \overline{\lambda(t)} > \beta' - 2m - l + \dfrac{n}{2}$ and $F \in H^{l,0}_{\beta'}(e^{-\gamma t}, K_\infty)$, so $c(t) \in L_{2,\mathrm{loc}}[0,\infty)$. Hence the assertion is proved for $h = 0$.

Assume that the assertion is true for $0, 1, \ldots, h-1$. Denoting u_{t^h} by v. From (2.1) we obtain

$$(-1)^{m-1} L_0(0,t,D)v = F_{t^h} + (-1)^m \sum_{k=1}^{h} \binom{h}{k} L_{0t^k}(0,t,D) u_{t^{h-k}}, \tag{3.4}$$

where

$$L_{0t^k} = \sum_{|p|=|q|=m} \frac{\partial^k a_{pq}(0,t)}{\partial t^k} D^p D^q.$$

Putting $S_0(\omega,t) = r^{-i\lambda(t)}\phi(\omega,t)$. Since $\phi(\omega,t) \in C^\infty(\omega,t)$ [21], from (3.3) it follows that

$$\sum_{k=1}^{h} \binom{h}{k} L_{0t^k}(0,t,D) u_{t^{h-k}} = \sum_{k=1}^{h} \binom{h}{k} L_{0t^k}(0,t,D) \big[(cS_0)_{t^{h-k}}\big] +$$

$$+ \sum_{k=1}^{h} \binom{h}{k} L_{0t^k}(0,t,D)(u_1)_{t^{h-k}}.$$

Using the induction hypothesis, we obtain

$$\sum_{k=1}^{h} \binom{h}{k} L_{0t^k}(0,t,D) u_{t^{h-k}} = F_1 - \sum_{k=1}^{h} \binom{h}{k} c_{t^{h-k}} L_0(0,t,D)(S_0)_{t^k}, \tag{3.5}$$

where $F_1 \in H^{l,0}_{\beta'}(e^{-\gamma_{h-1} t}, K_\infty)$. From (3.4) and (3.5) we see that

$$(-1)^{m-1} L_0(0,t,D)v = F_2 - (-1)^m \sum_{k=1}^{h} \binom{h}{k} c_{t^{h-k}} L_0(0,t,D)(S_0)_{t^k}, \tag{3.6}$$

where $F_2 \in H^{l,0}_{\beta'}(e^{-\gamma_h t}, K_\infty)$. Hence by arguments used in the proof of case $h = 0$ we can find

$$u_{t^h} = v = \sum_{k=1}^{h} \binom{h}{k} c_{t^{h-k}} (S_0)_{t^k} + d(t) S_0 + u_2, \tag{3.7}$$

where $d(t) \in L_{2,\mathrm{loc}}[0,\infty)$, $u_2 \in H^{2m+l,0}_{\beta'}(e^{-\gamma_h t}, K_\infty)$. From this equality it follows that

$$S_{0,1} = u_{t^h} - \sum_{k=2}^{h} \binom{h}{k} c_{t^{h-k}} (S_0)_{t^k} - (h-1) c_{t^{h-1}} (S_0)_t = c_{t^{h-1}} (S_0)_t + dS_0 + u_2. \tag{3.8}$$

Now differentiating the equality (3.3) $(h-1)$ times by t. As a result we obtain

$$u_{t^{h-1}} = \sum_{k=0}^{h-1} \binom{h-1}{k} c_{t^{h-k-1}} (S_0)_{t^k} + (u_1)_{t^{h-1}}. \tag{3.9}$$

We rewrite (3.9) in the form

$$S_{0,2} = u_{t^{h-1}} - \sum_{k=1}^{h-1} \binom{h-1}{k} c_{t^{h-k-1}} (S_0)_{t^k} = c_{t^{h-1}} S_0 + (u_1)_{t^{h-1}}. \tag{3.10}$$

Then

$$(S_{0,2})_t = u_{t^h} - \sum_{k=1}^{h-1} \binom{h-1}{k} \left[c_{t^{h-k}} (S_0)_{t^k} + c_{t^{h-k-1}} (S_0)_{t^{k+1}} \right] =$$

$$= u_{t^h} - \sum_{k=1}^{h} \binom{h}{k} c_{t^{h-k}} (S_0)_{t^k} + c_{t^{h-1}} (S_0)_t.$$

From this equality and (3.7) we obtain

$$(S_{0,2})_t = c_{t^{h-1}} (S_0)_t + dS_0 + u_2.$$

Putting $S_1 = S_0^{-1} (u_1)_{t^{h-1}}$, $S_2 = S_0^{-1} u_2 - S_0^{-2} (S_0)_t (u_1)_{t^{h-1}}$. It is easy to check that

$$S_0^{-1} S_{0,2} = c_{t^{h-1}} + S_1, \qquad (S_0^{-1} S_{0,2})_t = d + S_2.$$

It follows that

$$I(t) = c_{t^{h-1}}(t) - c_{t^{h-1}}(0) - \int_0^t d(\tau) d\tau =$$

$$= \int_0^t S_2(x, \tau) d\tau - S_1(x, t) + S_1(x, 0).$$

Since $(u_1)_{t^{h-1}} \in H_{\beta'}^{2m+l,0}(e^{-\gamma_{h-1} t}, K_\infty)$, $u_2 \in H_{\beta'}^{2m+l,0}(e^{-\gamma_h t}, K_\infty)$, so S_1, $S_2 \in$ $\in H_{-n/2}^{0,0}(e^{-\gamma_h t}, K_\infty)$. Therefore $I(t) \in H_{-\frac{n}{2}}^0(K)$, i.e., $I(t) \equiv 0$. Hence $c_{t^h} = d \in$ $\in L_{2,\mathrm{loc}}[0, \infty)$ and $(u_1)_{t^h} = u_2 \in H_{\beta'}^{2m+l,0}(e^{-\gamma_h t}, K_\infty)$.

This completes the proof.

Proposition 3.1. *Let* $u(x,t)$ *be a generalized solution of the problem* (1.1)–(1.3) *in the spaces* $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t}, \Omega_\infty)$ *such that* $u \equiv 0$ *whenever* $|x| > R = \mathrm{const}$, *and let* $f_{t^k} \in L^\infty(0, \infty; L_2(K))$ *for* $k \leqslant h+2$, $f_{t^k}(x, 0) = 0$ *for* $k \leqslant h+1$. *Assume that the straight lines*

$$\operatorname{Im} \lambda = m - \frac{n}{2} \quad \textit{and} \quad \operatorname{Im} \lambda = 2m - \frac{n}{2}$$

do not contain points of spectrum of the problem (2.2), (2.3) *for every* $t \in [0, \infty)$, *and in the strip*

$$m - \frac{n}{2} < \operatorname{Im} \lambda < 2m - \frac{n}{2}$$

there exists only one simple eigenvalue $\lambda(t)$ *of the problem* (2.2), (2.3). *Then the following representation holds*:

$$u(x,t) = \sum_{s=0}^{m-1} c_s(t) r^{-i\lambda(t)+s} P_{m-1,s}(\ln r) + u_1(x,t), \tag{3.11}$$

where $P_{m-1,s}$ *is a polynomial having order less than* m *and its coefficients are infinitely differentiable functions of* (ω, t), $(c_s)_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_1)_{t^k} \in H_0^{2m,0}(e^{-\gamma_{k+1}t}, K_\infty)$ *for* $k \leqslant h$.

Proof. First we will prove that if

$$m - \frac{n}{2} < \operatorname{Im} \lambda(t) < m + m_0 - \frac{n}{2}, \quad 1 \leqslant m_0 \leqslant m,$$

then

$$u(x,t) = \sum_{s=0}^{m_0-1} c_s(t) r^{-i\lambda(t)+s} P_{m_0-1,s}(\ln r) + u_1(x,t), \tag{3.12}$$

where $P_{m_0-1,s}$ is a polynomial having order less than m_0 and its coefficients are infinitely differentiable functions of (ω,t), $(c_s)_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$ and $(u_1)_{t^k} \in H^{2m,0}_{m-m_0}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h$.

We introduce the notation: $L_1 = (-1)^{m-1}\big[L_0(0,t,D) - L(x,t,D)\big]$. From the system (1.1) we get

$$(-1)^{m-1} L_0(0,t,D)u = F, \tag{3.13}$$

where $F = -i(u_t + f) + L_1 u$.

From Theorem 2.1 and Lemma 2.1 it follows that $u_{t^k} \in H^{2m,0}_m(e^{-\gamma_{k+1}t}, K_\infty)$, $k \leqslant h$. On the other hand, $u_{t^{k+1}} \in H^{m,0}(e^{-\gamma_{k+1}t}, K_\infty)$, $f_{t^k} \in L^\infty(0,\infty; L_2(K))$, $k \leqslant h$. Therefore $F_{t^k} \in H^{0,0}_{m-1}(e^{-\gamma_{k+1}t}, K_\infty), k \leqslant h$.

Let $m - \dfrac{n}{2} < \operatorname{Im} \lambda(t) < m + 1 - \dfrac{n}{2}$. From Lemma 3.2 it follows that

$$u(x,t) = c(t) r^{-i\lambda(t)} \phi(\omega,t) + u_1(x,t), \tag{3.14}$$

where ϕ is an infinitely differentiable function of (ω,t) what does not depend on the solution, $c_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_1)_{t^k} \in H^{2m,0}_{m-1}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h$. Hence (3.12) is proved for $m_0 = 1$.

Assume that (3.12) holds for $m_0 \leqslant m-1$. We distinguish the following cases.

Case 1: $m - \dfrac{n}{2} < \operatorname{Im} \lambda(t) < m + m_0 - \dfrac{n}{2}$. Using the induction hypothesis we obtain (3.12). Putting

$$S_{m_0} = (-1)^m \sum_{s=0}^{m_0-1} c_s(t) r^{-i\lambda(t)+s} P_{m_0-1,s}(\ln r). \tag{3.15}$$

Then

$$LS_{m_0} = F_1(x,t) + \sum_{j+s \leqslant m_0} \sum_{s=0}^{m_0-1} c_s(t) r^{-i\lambda(t)-2m+s+j} \widetilde{P}_{m_0-1,s,j}(\ln r), \tag{3.16}$$

where $(F_1)_{t^k} \in H^{0,0}_{m-m_0-1}(e^{-\gamma t}, K_\infty)$ for $k \leqslant h$, and $\widetilde{P}_{m_0-1,s,j}$ is a polynomial having order less than m_0 and its coefficients are infinitely differentiable functions of (ω, t).

From (3.12), (3.13), and (3.16) we obtain

$$(-1)^{m-1} L_0(0,t,D)u_1 = F_2(x,t)+$$
$$+\sum_{j+s\leqslant m_0} \sum_{s=0}^{m_0-1} c_s(t) r^{-i\lambda(t)-2m+s+j} \widetilde{P}_{m_0-1,s,j}(\ln r), \tag{3.17}$$

where $F_2 = -i(u_t+f) + L_1 u_1 + F_1 \in H^{0,0}_{m-m_0-1}(e^{-\gamma_1 t}, K_\infty)$.

Since $(u_1)_{t^k} \in H^{2m,0}_{m-m_0}(e^{-\gamma_{k+1} t}, K_\infty)$ for $k \leqslant h$, we have

$$(F_2)_{t^k} \in H^{0,0}_{m-m_0-1}(e^{-\gamma_{k+1} t}, K_\infty),$$

for $k \leqslant h$.

By Lemma 3.1 there exists a function

$$\omega_1 = \sum_{j+s\leqslant m_0} \sum_{s=0}^{m_0-1} c_s(t) r^{-i\lambda(t)+s+j} P_{m_0,s,j}(\ln r) \tag{3.18}$$

such that

$$(-1)^{m-1} L_0(0,t,D)\omega_1 = \sum_{j+s\leqslant m_0} \sum_{s=0}^{m_0-1} c_s(t) r^{-i\lambda(t)-2m+s+j} \widetilde{P}_{m_0-1,s,j}(\ln r), \tag{3.19}$$

where $P_{m_0,s,j}$ is a polynomial having order less than m_0+1 and its coefficients are infinitely differentiable functions of (ω, t).

Putting $v_1 = u_1 - \omega_1$. From (3.17) and (3.19) it follows that

$$(-1)^{m-1} L_0(0,t,D)v_1 = F_2(x,t).$$

By Lemma 3.2 we obtain

$$v_1(x,t) = c(t) r^{-i\lambda(t)} \varphi(\omega,t) + u_2(x,t), \tag{3.20}$$

where φ is an infinitely differentiable function of (ω, t) which does not depend on the solution, $c_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_2)_{t^k} \in H^{2m,0}_{m-m_0-1}(e^{-\gamma_{k+1} t}, K_\infty)$ for $k \leqslant h$.

From (3.18) and (3.20) it follows that

$$u_1(x,t) = \sum_{j+s\leqslant m_0} \sum_{s=0}^{m_0-1} c_s(t) r^{-i\lambda(t)+s+j} P_{m_0,s,j}(\ln r)+$$
$$+\, c(t) r^{-i\lambda(t)} \varphi(\omega,t) + u_2(x,t).$$

Hence and from (3.12) we get

$$u(x,t) = \sum_{s=0}^{m_0} \widetilde{c}_s(t) r^{-i\lambda(t)+s} \widetilde{P}_{m_0,s}(\ln r) + u_2(x,t), \tag{3.21}$$

where $\widetilde{P}_{m_0,s}$ is a polynomial having order less than m_0+1 and its coefficients are infinitely differentiable functions of (ω,t), $(\widetilde{c}_s)_{t^k}\in L_{2,\mathrm{loc}}[0,\infty)$, $(u_2)_{t^k}\in H^{2m,0}_{m-m_0-1}(e^{-\gamma_{k+1}t}, K_\infty)$ for $0\leqslant k\leqslant h$.

Case 2: $m+m_0-\dfrac{n}{2}<\operatorname{Im}\lambda(t)<m+m_0+1-\dfrac{n}{2}$. Since the strip $m-\dfrac{n}{2}\leqslant$ $\leqslant\operatorname{Im}\lambda\leqslant m+m_0-\dfrac{n}{2}$ does not contain points of spectrum of the problem (2.2), (2.3) so $u_{t^k}\in H^{2m,0}_{m-m_0}(e^{-\gamma_{k+1}t},K_\infty), k\leqslant h$. Therefore, from Lemma 3.2 it follows that

$$u(x,t)=c(t)r^{-i\lambda(t)}\varphi(\omega,t)+u_1(x,t),$$

where φ is an infinitely differentiable function of (ω,t) which does not depend on the solution , $c_{t^k}\in L_{2,\mathrm{loc}}[0,\infty)$, $(u_1)_{t^k}\in H^{2m,0}_{m-m_0-1}(e^{-\gamma_{k+1}t},K_\infty)$ for $k\leqslant h$.

Case 3: There exists t_0 such that $\operatorname{Im}\lambda(t_0)=m+m_0-\dfrac{n}{2}$. Dividing the interval $[0,\infty)$ by points $T_0=0<T_1<\ldots<T_s<\ldots$ such that one of following cases happens in each interval $[T_{s-1},T_s], s=1,2,\ldots$:

(i) $m-\dfrac{n}{2}<\operatorname{Im}\lambda(t)<m+m_0-\dfrac{n}{2}$,

(ii) $m+m_0-\dfrac{n}{2}<\operatorname{Im}\lambda(t)<m+m_0+1-\dfrac{n}{2}$,

(iii) $m+m_0-\mu-\dfrac{n}{2}<\operatorname{Im}\lambda(t)<m+m_0-\mu+1-\dfrac{n}{2}, 0<\mu<1$.

If (i) (or (ii)) happens in the interval $[T_{s-1},T_s]$, then by repeating the proof of Case 1 (resp. Case 2), we obtain

$$u(x,t)=c^{(s)}(t)r^{-i\lambda(t)}\varphi(\omega,t)+u_1^{(s)}(x,t),\quad t\in[T_{s-1},T_s],\tag{3.22}$$

where φ is an infinitely differentiable function of (ω,t) what does not depend on the solution, $c^{(s)}_{t^k}\in L_2[T_{s-1},T_s]$, $(u_1)^{(s)}_{t^k}\in H^{2m,0}_{m-m_0-1}(K\times[T_{s-1},T_s])$ for $k\leqslant h$. If (iii) happens then repeating the argument in the proof of Case 2, we obtain (3.22) for $(u_1^{(s)})_{t^k}\in H^{2m,0}_{m-m_0-1+\mu}(K\times[T_{s-1},T_s]), k\leqslant h$. Hence and from arguments analogous to the proof of Case 1, after that set $c(t)=c^{(s)}(t)$, $u_1(x,t)=u_1^{(s)}(x,t)$, whenever $t\in[T_{s-1},T_s]$, we obtain (3.21).

From above arguments follow (3.12). For $m_0=m$, from (3.12) we obtain (3.11).

Proposition 3.1 is proved.

Proposition 3.2. *Let $u(x,t)$ be a generalized solution of the problem* (1.1)–(1.3) *in the spaces $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty)$ such that $u\equiv 0$ whenever $|x|>R=\mathrm{const}$, and let $f_{t^k}\in L^\infty(0,\infty;H^l_0(K))$ for $k\leqslant 2l+h+2$, $f_{t^k}(x,0)=0$ for $k\leqslant 2l+h+1$. Assume that the straight lines*

$$\operatorname{Im}\lambda=m-\frac{n}{2}\quad\textit{and}\quad\operatorname{Im}\lambda=2m+l-\frac{n}{2}$$

do not contain points of spectrum of the problem (2.2), (2.3) *for every $t\in[0,\infty)$, and in the strip*

$$m-\frac{n}{2}<\operatorname{Im}\lambda<2m+l-\frac{n}{2}$$

there exists only one simple eigenvalue $\lambda(t)$ of the problem (2.2), (2.3). *Then the following representation holds*:

$$u(x,t) = \sum_{s=0}^{l+m-1} c_s(t) r^{-i\lambda(t)+s} P_{3l+m-1,s}(\ln r) + u_1(x,t), \tag{3.23}$$

where $P_{3l+m-1,s}$ is a polynomial having order less than $3l+m$ and its coefficients are infinitely differentiable functions of (ω,t), $(c_s)_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_1)_{t^k} \in \in H_0^{2m+l,0}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h+l$.

Proof. We will use the induction on l. If $l=0$ the statement follows from Proposition 3.1. Let the statement be true for $l-1$. We distinguish the following cases:

Case 1: $m - \dfrac{n}{2} < \operatorname{Im}\lambda(t) < 2m+l-1-\dfrac{n}{2}$. From inductive hypothesis we obtain

$$u(x,t) = \sum_{s=0}^{l+m-2} c_s(t) r^{-i\lambda(t)+s} P_{3l+m-4,s}(\ln r) + u_1(x,t), \tag{3.24}$$

where $P_{3l+m-4,s}$ is a polynomial having order less than $3l+m-3$ and its coefficients are infinitely differentiable functions of (ω,t), $(c_s)_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_1)_{t^k} \in \in H_0^{2m+l-1,0}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h+l-1$.

From (3.13) and (3.24) we find

$$(-1)^{m-1} L_0(0,t,D) u_1 = F_3 + (-1)^m LS - iS_t, \tag{3.25}$$

where $F_3 = -i[(u_1)_t + f] + L_1 u_1$, and

$$S = \sum_{s=0}^{l+m-2} c_s(t) r^{-i\lambda(t)+s} P_{3l+m-4,s}(\ln r).$$

Since $f_{t^k} \in L^\infty(0,\infty; H_0^l(K))$ for $k \leqslant 2l+h+2$ and $f_{t^k}(x,0) = 0$ for $k \leqslant \leqslant 2l+h+1$, so $f_{t^k} \in L^\infty(0,\infty; H_0^{l-1}(K))$, $k \leqslant 2(l-1)+(h+2)+2$, and $f_{t^k}(x,0) = 0$, $k \leqslant 2(l-1)+(h+2)+1$. Therefore, $(c_s)_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$ and $(u_1)_{t^k} \in H_0^{2m+l-1,0}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h+l+1$. Hence it follows that $(F_3)_{t^k} \in \in H_0^{l,0}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h+l$. On the other hand

$$(-1)^m LS - iS_t = F_4 + \sum_{s=0}^{l-1+m} \widetilde{c}_s(t) r^{-i\lambda(t)-2m+s} \widetilde{P}_{3l+m-2,s}(\ln r),$$

where $\widetilde{P}_{3l+m-2,s}$ is a polynomial having order less than $3l+m-1$ and its coefficients are infinitely differentiable functions of (ω,t), $(F_4)_{t^k} \in H_0^{l,0}(e^{-\gamma_{k+1}t}, K_\infty)$, and $(\widetilde{c}_s)_{t^k} \in \in L_{2,\mathrm{loc}}[0,\infty)$ for $k \leqslant h+l$. Therefore from (3.25) we obtain

$$(-1)^{m-1} L_0(0,t,D) u_1 = F_5 + \sum_{s=0}^{l+m-1} \widetilde{c}_s(t) r^{-i\lambda(t)-2m+s} \widetilde{P}_{3l+m-2,s}(\ln r), \tag{3.26}$$

where $F_5 = F_3 + F_4 \in H_0^{l,0}(e^{-\gamma_1 t}, K_\infty) \subseteq H_{-1}^{l-1,0}(e^{-\gamma_1 t}, K_\infty)$.

By Lemma 3.2 and by arguments used in the proof of Proposition 3.1 we can find

$$u_1(x,t) = \sum_{s=0}^{l+m-1} \widetilde{c}_s(t) r^{-i\lambda(t)+s} \widetilde{P}_{3l+m-1,s}(\ln r) + u_2(x,t), \tag{3.27}$$

where $\widetilde{P}_{3l+m-1,s}$ is a polynomial having order less than $3l+m$ and its coefficients are infinitely differentiable functions of (ω,t), $(u_2)_{t^k} \in H^{2m+l-1,0}_{-1}(e^{-\gamma_{k+1}t},K_\infty)$ for $k \leqslant h+l$. By Lemma 2.2 we have $(u_2)_{t^k} \in H^{2m+l,0}_0(e^{-\gamma_{k+1}t},K_\infty)$ for $k \leqslant h+l$. Hence and from (3.24) it follows that

$$u(x,t) = \sum_{s=0}^{l+m-1} c_s(t) r^{-i\lambda(t)+s} P_{3l+m-1,s}(\ln r) + u_2(x,t), \tag{3.28}$$

where $P_{3l+m-1,s}$ is a polynomial having order less than $3l+m$ and its coefficients are infinitely differentiable functions of (ω,t), $(c_s)_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, and $(u_2)_{t^k} \in \in H^{2m+l,0}_0(e^{-\gamma_{k+1}t},K_\infty)$ for $k \leqslant h+l$.

Case 2: $2m+l-1-\dfrac{n}{2} < \operatorname{Im}\lambda(t) < 2m+l-\dfrac{n}{2}$. It follows from Theorem 2.1 and Lemma 2.1 that $u_{t^k} \in H^{2m,0}_m(e^{-\gamma_{k+1}t},K_\infty)$ for $k \leqslant h+2l$. On the other hand, the strip $m-\dfrac{n}{2} \leqslant \operatorname{Im}\lambda \leqslant 2m-\dfrac{m}{2}$ does not contain points of spectrum of the problem (2.2), (2.3) for every $t\in[0,\infty)$. Hence and from theorems on the smoothness of solutions of elliptic problems in domains with conical points (see, e.g., [4, 5, 8, 22]) it follows that $u_{t^k} \in H^{2m,0}_0(e^{-\gamma_{k+1}t},K_\infty)$ for $k \leqslant h+2l$.

We will prove that if $f_{t^k} \in L^\infty(0,\infty;H^j_0(K))$ for $k \leqslant 2j+h+2$ and $f_{t^k}(x,0)=0$ for $k \leqslant 2j+h+1$, then $u_{t^k} \in H^{2m+j,0}_0(e^{-\gamma_{k+1}t},K_\infty)$, $k \leqslant h+2l-j$. This assertion was proved for $j=0$. Assume that it is true for $j-1$. Since $f_{t^k} \in L^\infty(0,\infty;H^{j-1}_0(K))$ for $k \leqslant 2(j-1)+(h+2)+2$ and $f_{t^k}(x,0)=0$ for $k \leqslant 2(j-1)+(h+2)+1$, then from inductive hypothesis it follows that $u_{t^k} \in H^{2m+j-1,0}_0(e^{-\gamma_{k+1}t},K_\infty)$, $k \leqslant h+2l-j+3$. Therefore $u_{t^{k+1}} \in H^{j-1,0}_{-1}(e^{-\gamma_{k+2}t},K_\infty)$ for $k \leqslant h+2l-j$. Hence and from the fact that the strip

$$2m+j-1-\frac{n}{2} \leqslant \operatorname{Im}\lambda \leqslant 2m+j-\frac{n}{2}$$

does not contain points of spectrum of the problem (2.2), (2.3) for every $t\in[0,\infty)$, we obtain $u_{t^k} \in H^{2m+j-1,0}_{-1}(e^{-\gamma_{k+1}t},K_\infty)$, $k \leqslant h+2l-j$. It follows from Lemma 2.2 that $u_{t^k} \in H^{2m+j,0}_0(e^{-\gamma_{k+1}t},K_\infty)$ for $k \leqslant h+2l-j$.

By Lemma 3.2 and from above arguments we obtain

$$u(x,t) = c(t) r^{-i\lambda(t)} \varphi(\omega,t) + u_1(x,t), \tag{3.29}$$

where φ is an infinitely differentiable function of (ω,t) which does not depend on the solution, $c_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, and $(u_1)_{t^k} \in H^{2m+l,0}_0(e^{-\gamma_{k+1}t},K_\infty)$ for $k \leqslant h+l$.

Case 3: There exists t_0 such that $\operatorname{Im}\lambda(t_0) = 2m+l-1-\dfrac{n}{2}\cdot$ By arguments used in Case 3 in the proof of Proposition 3.1, this case can be managed.

Proposition 3.2 is proved.

Proposition 3.3. *Let* $u(x,t)$ *be a generalized solution of the problem* (1.1)–(1.3) *in the spaces* $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t},\Omega_\infty)$ *such that* $u\equiv 0$ *whenever* $|x|>R=\mathrm{const}$, *and let* $f_{t^k} \in L^\infty(0,\infty;H^l_0(K))$ *for* $k \leqslant 2l+h+2$, $f_{t^k}(x,0)=0$ *for* $k \leqslant 2l+h+1$. *Assume that the straight lines*

$$\operatorname{Im}\lambda = m-\frac{n}{2} \quad \textit{and} \quad \operatorname{Im}\lambda = 2m+l-\frac{n}{2}$$

do not contain points of spectrum of the problem (2.2), (2.3) *for every* $t\in[0,\infty)$, *and in the strip*

$$m - \frac{n}{2} < \operatorname{Im}\lambda < 2m + l - \frac{n}{2}$$

there exist only simple eigenvalues $\lambda_1(t), \lambda_2(t), \ldots, \lambda_{N_0}(t)$ *of the problem* (2.2), (2.3) *such that*

$$\operatorname{Im}\lambda_1(t) < \operatorname{Im}\lambda_2(t) < \ldots < \operatorname{Im}\lambda_{N_0}(t), \quad t \in [0, \infty),$$

$$\operatorname{Im}\lambda_j(t) \neq \operatorname{Im}\lambda_k(t) + N, \quad j \neq k, \quad N \in \mathbb{Z}, \quad j, k = 1, \ldots, N_0.$$

Then the following representation holds:

$$u(x,t) = \sum_{j=1}^{N_0} \sum_{s=0}^{l+m-1} c_{s,j}(t) r^{-i\lambda_j(t)+s} P_{3l+m-1,s,j}(\ln r) + u_1(x,t), \tag{3.30}$$

where $P_{3l+m-1,s,j}$ *is a polynomial having order less than* $3l + m$ *and its coefficients are infinitely differentiable functions of* (ω, t), $(c_{s,j})_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_1)_{t^k} \in \in H_0^{2m+l,0}(e^{-\gamma_{k+1}t}, K_\infty)$ *for* $k \leqslant h + l$.

Proof. For each $t_0 \in (0,\infty)$ there exists $\varepsilon > 0$ such that $m + \mu_{j-1} - \dfrac{n}{2} < \operatorname{Im}\lambda_j(t) < < m + \mu_j - \dfrac{n}{2}$, $t \in [t_0 - \varepsilon, t_0 + \varepsilon]$, $\mu_j = \text{const} \geq 0$, $j = 1, \ldots, N_0$. Therefore, there exist the numbers $T_0 = 0 < T_1 < T_2 < \ldots$ such that $m + \mu_{j-1,s} - \dfrac{n}{2} < \operatorname{Im}\lambda_j(t) < < m + \mu_{j,s} - \dfrac{n}{2}$, $t \in [T_{s-1}, T_s]$, $\mu_{j,s} = \text{const}$, $j = 1, \ldots, N_0$, $s = 1, 2, \ldots$. By arguments used in case 3 in the proof of Proposition 3.1 if necessary, we can assume that

$$m - \frac{n}{2} < \operatorname{Im}\lambda_1(t) < m + \mu_1 - \frac{n}{2} < \operatorname{Im}\lambda_2(t) < \ldots$$

$$\ldots < m + \mu_{N_0-1} - \frac{n}{2} < \operatorname{Im}\lambda_{N_0}(t) < 2m + l - \frac{n}{2}, \quad t \in [0, \infty).$$

In order to prove the statement we will use induction on N_0. If $N_0 = 1$ the statement follows from Proposition 3.2. Let the statement be true for $N_0 - 1$. First, consider the case $\mu_{N_0-1} \geq m$. For simplicity we assume that $\mu_{N_0-1} = m + l_0$, $l_0 < l$. From inductive hypothesis we obtain

$$u(x,t) = \sum_{j=1}^{N_0-1} \sum_{s=0}^{l_0+m-1} c_{s,j}(t) r^{-i\lambda_j(t)+s} P_{3l_0+m-1,s,j}(\ln r) + u_1(x,t), \tag{3.31}$$

where $P_{3l_0+m-1,s,j}$ is a polynomial having order less than $3l_0 + m$ and its coefficients are infinitely differentiable functions of (ω, t), $(c_{s,j})_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_1)_{t^k} \in \in H_0^{2m+l_0,0}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h + l_0$. Repeating arguments analogous to the proof of (3.26) we have

$$(-1)^{m-1} L_0(0,t,D) u_1 = \widetilde{F} + \sum_{j=1}^{N_0-1} \sum_{s=0}^{l_0+m} \widetilde{c}_{s,j}(t) r^{-i\lambda_j(t)-2m+s} \widetilde{P}_{3l_0+m+1,s,j}(\ln r), \tag{3.32}$$

where $\widetilde{F} \in H_0^{l_0+1,0}(e^{-\gamma_1 t}, K_\infty)$, $\widetilde{P}_{3l_0+m+1,s,j}$ is a polynomial having order less than $3l_0 + m + 2$ and its coefficients are infinitely differentiable functions of (ω, t), $(\widetilde{c}_{s,j})_{t^k} \in$

$\in L_{2,\mathrm{loc}}[0,\infty),\ k \leqslant h$. Hence it follows that if $2m + l_1 - \dfrac{n}{2} < \operatorname{Im}\lambda_{N_0}(t) < 2m + l_1 + + 1 - \dfrac{n}{2}$ for $l_1 \geq l_0$, then

$$u_1(x,t) = \sum_{j=1}^{N_0} \sum_{s=0}^{l_1+m} \widetilde{c}_{s,j}(t) r^{-i\lambda_j(t)+s} \widetilde{P}_{3l_1+m+2,s}(\ln r) + u_2(x,t), \tag{3.33}$$

where $\widetilde{P}_{3l_1+m+2,s}$ is a polynomial having order less than $3l_1 + m + 3$ and its coefficient are infinitely differentiable functions of (ω, t), $(u_2)_{t^k} \in H_0^{2m+l_1+1,0}(e^{-\gamma_{k+1}t}, K_\infty)$, for $k \leqslant h + l_1$.

Since the strip

$$2m + l_1 + 1 - \frac{n}{2} \leqslant \operatorname{Im}\lambda \leqslant 2m + l - \frac{n}{2}$$

does not contain points of spectrum of the problem (2.2), (2.3) so from (3.31), (3.32) and (3.33) we obtain (3.30).

If there exists t_0 such that $\operatorname{Im}\lambda_{N_0}(t_0) = 2m + l_1 - \dfrac{n}{2}$ then by Lemma 3.2 and from arguments used in case 3 in the proof of Proposition 3.1 we obtain (3.30).

Finally, if $\mu_{N_0-1} < m$, for simplicity we assume that $\mu_{N_0-1} = m_0, 0 \leqslant m_0 < m$, then repeating the proofs of Proposition 3.1, Proposition 3.2, using above arguments and Lemma 3.2 we obtain the conclusion.

Proposition 3.3 is proved.

We can now state the main theorem on the asymptotic expansion of the generalized solution of problem (1.1)–(1.3) in a neighbourhood of the conical point.

Theorem 3.2. *Let $u(x,t)$ be a generalized solution of the problem* (1.1)–(1.3) *in the spaces $\overset{\circ}{H}{}^{m,0}(e^{-\gamma t}, \Omega_\infty)$, and let $f_{t^k} \in L^\infty(0,\infty; H_0^l(\Omega))$ for $k \leqslant 2l + h + 2$, $f_{t^k}(x,0) = 0$ for $k \leqslant 2l + h + 1$. Assume that the straight lines*

$$\operatorname{Im}\lambda = m - \frac{n}{2} \quad \text{and} \quad \operatorname{Im}\lambda = 2m + l - \frac{n}{2}$$

do not contain points of spectrum of the problem (2.2), (2.3) *for every $t \in [0,\infty)$, and in the strip*

$$m - \frac{n}{2} < \operatorname{Im}\lambda < 2m + l - \frac{n}{2}$$

there exist only simple eigenvalues $\lambda_1(t), \ldots, \lambda_{N_0}(t)$ of the problem (2.2), (2.3) *such that*

$$\operatorname{Im}\lambda_1(t) < \operatorname{Im}\lambda_2(t) < \ldots < \operatorname{Im}\lambda_{N_0}(t), \quad t \in [0,\infty),$$

$$\operatorname{Im}\lambda_j(t) \neq \operatorname{Im}\lambda_k(t) + N, \quad j \neq k, \quad N \in \mathbb{Z}, \quad j,k = 1,\ldots,N_0.$$

Then the following representation holds in a neighbourhood of the conical point

$$u(x,t) = \sum_{j=1}^{N_0} \sum_{s=0}^{l+m-1} c_{s,j}(t) r^{-i\lambda_j(t)+s} P_{3l+m-1,s,j}(\ln r) + u_1(x,t), \tag{3.34}$$

where $P_{3l+m-1,s,j}$ is a polynomial having order less than $3l + m$ and its coefficients are infinitely differentiable functions of (ω, t), $(c_{s,j})_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$ and $(u_1)_{t^k} \in \in H_0^{2m+l,0}(e^{-\gamma_{k+1}t}, \Omega_\infty)$ for $k \leqslant h + l$.

Proof. Surrounding the point 0 by a neighbourhood U_0 with so small diameter that the intersection of Ω and U_0 coincides with K. Consider a function $u_0 = \varphi_0 u$, where $\varphi_0 \in \overset{\circ}{C}{}^{\infty}(U_0)$ and $\varphi_0 \equiv 1$ in some neighbourhood of 0. The function u_0 satisfies the system

$$(-1)^{m-1} iL(x,t,D)u_0 - (u_0)_t = \varphi_0 f + L'(x,t,D)u,$$

where $L'(x,t,D)$ is a linear differential operator having order less than $2m$. Coefficients of this operator depend on the choice of the function φ_0 and equal to 0 outside U_0. Hence and from arguments analogous to the proof of Proposition 3.3, we obtain

$$\varphi_0 u(x,t) = \sum_{j=1}^{N_0} \sum_{s=0}^{l+m-1} c_{s,j}(t) r^{-i\lambda_j(t)+s} P_{3l+m-1,s,j}(\ln r) + u_2(x,t), \tag{3.35}$$

where $P_{3l+m-1,s,j}$ is a polynomial having order less than $3l+m$ and its coefficients are infinitely differentiable functions of (ω, t), $(c_{s,j})_{t^k} \in L_{2,\mathrm{loc}}[0,\infty)$, $(u_2)_{t^k} \in H_0^{2m+l,0}(e^{-\gamma_{k+1}t}, K_\infty)$ for $k \leqslant h+l$.

The function $\varphi_1 u = (1-\varphi_0)u$ equals to 0 in some neighbourhood of the conical point. We can apply the known theorem on the smoothness of solutions of elliptic problems in a smooth domain to this function and obtain $\varphi_1 u \in H_0^{2m+l}(\Omega)$ for a.e. $t \in [0,\infty)$. Hence we have $(\varphi_1 u)_{t^k} \in H_0^{2m+l,0}(e^{-\gamma_{k+1}t}, \Omega_\infty)$ for $k \leqslant h+l$.

Since $u = \varphi_0 u + \varphi_1 u$ so from (3.35) we obtain (3.34).

Theorem 3.2 is proved.

4. An example. The Schrödinger equation is the fundamental equation of nonrelativistic quantum mechanics. In the simplest case for a particle without spin in an external field it has the form

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar}{2m} \triangle \psi + V(x)\psi,$$

where $x \in \mathbb{R}^3$, $\psi = \psi(x,t)$ is the wave function of a quantum particle, giving the complex amplitude characterizing the presence of the particle at each point x (in particular $|\psi(x,t)|^2$ is interpreted as the probability density for the particle to be at the point x at the instant t), m is the mass of the particle, $\hbar$ is Planck's constant, and $V(x)$ is the external field potential (a real-value function).

We consider in Ω_∞ the mathematical model of Schrödinger equation

$$i\triangle u - u_t = f, \tag{4.1}$$

with an initial condition

$$u|_{t=0} = 0, \tag{4.2}$$

and a boundary condition

$$u|_{S_\infty} = 0. \tag{4.3}$$

Let the cone $K = \{x\colon x/|x| \in G\}$, where G is a smooth domain on the unit sphere S^{n-1}. The Laplacian in polar coordinate (r,ω) in $\mathbb{R}^n$ is given by

$$(\triangle u)(r,\omega) = \frac{1}{r^{n-1}} \frac{\partial}{\partial r} \left(r^{n-1} \frac{\partial}{\partial r} \right) u(r,\omega) + \frac{1}{r^2} \triangle_\omega u(r,\omega),$$

where $\triangle_\omega$ is the Laplace – Beltrami operator on the unit sphere S^{n-1}. Therefore the corresponding spectral problem has the form

$$\triangle_\omega v + [(i\lambda)^2 + i(2-n)\lambda]v = 0, \quad \omega \in G, \tag{4.4}$$

$$v|_{\partial G} = 0. \tag{4.5}$$

4.1. Case $n = 2$. Assume in a neighbourhood of the coordinate origin, $\partial\Omega$ coincides with a rectilinear angle having measure is β. Then the spectral problem (4.4), (4.5) has the form

$$v_{\omega\omega} - \lambda^2 v = 0, 0 < \omega < \beta, \tag{4.6}$$

$$v(0) = v(\beta) = 0. \tag{4.7}$$

Eigenvalues of the problem (4.6), (4.7) are $\lambda_k = \pm\dfrac{i\pi k}{\beta}$, $k \in \mathbb{N}^*$. From Theorems 2.2 and 3.1 we obtain the following proposition.

Proposition 4.1. *Let $u(x,t)$ be a generalized solution of the problem* (4.1) – (4.3) *in the space $\overset{\circ}{H}{}^{1,0}(e^{-\gamma t}, \Omega_\infty)$, and let $f, f_t, f_{tt}, f_{ttt} \in L^\infty(0,\infty; L_2(\Omega))$, $f(x,0) = f_t(x,0) = f_{tt}(x,0) = 0$. Then*

(i) *if $\beta < \pi$, then $u \in H_0^2(e^{-\gamma_2 t}, \Omega_\infty)$,*

(ii) *if $\beta > \pi$, then*

$$u(x,t) = c(x,t)r^{\pi/\beta} + u_1(x,t),$$

where $c(x,t) \in V^2_{\pi/\beta}(e^{-\gamma_2 t}, \Omega_\infty)$, $u_1 \in H_0^2(e^{-\gamma_2 t}, \Omega_\infty)$.

4.2. Case $n = 3$. Let k_j, $j = 1, 2, \ldots$, are eigenvalues of the Dirichlet problem for the equation

$$\triangle_\omega v + kv = 0, \quad \omega \in G, \tag{4.8}$$

with $0 < k_1 \leqslant k_2 \leqslant k_3 \leqslant \ldots$. Then

$$\lambda_j = i\left(-\frac{1}{2} \pm \sqrt{\frac{1}{4} + k_j}\right), \quad j = 1, 2, \ldots, \tag{4.9}$$

are eigenvalues of the spectral problem (4.4), (4.5).

Let v_j be the eigenfunction corresponding to the eigenvalue λ_j.

We will prove that λ_j is simple. Indeed, let $\varphi_1, \varphi_2, \ldots, \varphi_r$, $r \geq 1$, are connecting functions of the eigenfunction v_j. From (4.4) and definition of the connecting function, we have

$$\triangle_\omega \bar{v}_j + [(i\lambda_j)^2 - i\lambda_j]\bar{v}_j = 0, \tag{4.10}$$

$$\triangle_\omega \varphi_l + [(i\lambda_j)^2 - i\lambda_j]\varphi_l - (2\lambda_j + i)v_j = 0. \tag{4.11}$$

Then

$$\int_G \triangle_\omega \varphi_l \bar{v}_j d\omega = \int_G \varphi_l \triangle_\omega \bar{v}_j d\omega. \tag{4.12}$$

From (4.10), (4.11) and (4.12) it follows that

$$(2\lambda_j + i) \int\limits_G |v_j|^2 d\omega = 0. \tag{4.13}$$

Since Im $(2\lambda_j + i) \neq 0$, so $v_j = 0$. It is a contradiction. Thus, λ_j is simple.

Let $f, f_t, f_{tt}, f_{ttt} \in L^\infty(0, \infty; L_2(\Omega), f(x, 0) = f_t(x, 0) = f_{tt}(x, 0) = 0$, and $u(x, t)$ be a generalized solution of the problem (4.1) – (4.3) in the space $\overset{\circ}{H}{}^{1,0}(e^{-\gamma t}, \Omega_\infty)$. We distinguish the following cases.

Case 1: $\operatorname{Im}\lambda_1 > \dfrac{1}{2}$. Since the strip $-\dfrac{1}{2} \leqslant \operatorname{Im}\lambda \leqslant \dfrac{1}{2}$ does not contain eigenvalues of the spectral problem (4.4), (4.5), from Theorem 2.2 we obtain that $u(x, t) \in \in H_0^2(e^{-\gamma_2 t}, \Omega_\infty)$.

Case 2: $\operatorname{Im}\lambda_1 \leqslant \dfrac{1}{2}$. Let $\lambda_1, \ldots, \lambda_{N_0}$, are eigenvalues of the spectral problem (4.4), (4.5) satisfying

$$-\frac{1}{2} < \operatorname{Im}\lambda_1 < \ldots < \operatorname{Im}\lambda_{N_0} \leqslant \frac{1}{2}.$$

(i) If the straight line $\operatorname{Im}\lambda = \dfrac{1}{2}$ does not contain eigenvalues of the spectral problem (4.4), (4.5), then from Theorem 3.2 we obtain

$$u(x, t) = \sum_{j=1}^{N_0} c_j(t) r^{\operatorname{Im}\lambda_j} \phi_j(\omega) + u_0(x, t), \tag{4.14}$$

where ϕ_j are infinitely differetiable functions of ω, and $c_j \in L_{2,\mathrm{loc}}[0, \infty)$, $j = 1, \ldots, N_0$, and $u_0 \in H_0^{2,0}(e^{-\gamma_1 t}, \Omega_\infty)$.

Consider the domain

$$\Omega^\rho = \Big\{x \in \Omega\colon \frac{1}{2}\rho < |x| < 2\rho\Big\}, \quad \rho = \text{const} > 0.$$

Let ρ be small enough such that the boundary of Ω^ρ coincides with the cone K. Putting $v(x', t) = u_0(\rho x', t)$. Since $u_0 \in H_0^{2,0}(e^{-\gamma_1 t}, \Omega_\infty)$, from embedding theorems for the domain $K' = \Big\{x' \in K\colon \dfrac{1}{2} < |x'| < 2\Big\}$, we obtain

$$|v(x', t)|^2 \leqslant C_1 \int\limits_{K'} \left(v^2 + |\operatorname{grad} v|^2 + \sum_{|\alpha|=2} |D^\alpha v|^2 \right) dx', \quad C_1 = \text{const}.$$

Substituting $x = \rho x'$ in this inequality, we obtain

$$|u_0(x, t)|^2 \leqslant C_1 \int\limits_{\Omega^\rho} \left(\rho^{-3} u_0^2 + \rho^{-1} |\operatorname{grad} u_0|^2 + \rho \sum_{|\alpha|=2} |D^\alpha u_0|^2 \right) dx.$$

Hence

$$\rho^{-1} |u_0(x, t)|^2 \leqslant C_1 \int\limits_{\Omega^\rho} \left(\rho^{-4} u_0^2 + \rho^{-2} |\operatorname{grad} u_0|^2 + \sum_{|\alpha|=2} |D^\alpha u_0|^2 \right) dx \leqslant$$

$$\leqslant C_2 \int\limits_{\Omega^\rho} \left(r^{-4} u_0^2 + r^{-2} |\operatorname{grad} u_0|^2 + \sum_{|\alpha|=2} |D^\alpha u_0|^2 \right) dx \leqslant$$

$$\leqslant C_3 \|u_0(x,t)\|^2_{H^2_0(\Omega)} \leqslant C_4 \|f(x,t)\|^2_{L_2(\Omega)},$$

where $C_i = \text{const}$, $i = 1, 2, 3, 4$. For $|x| = \rho$ we obtain

$$|u_0(x,t)| \leqslant C r^{1/2}, \quad C = \text{const}. \tag{4.15}$$

From (4.12) and (4.13) we have

$$|u(x,t)| \leqslant C r^{\operatorname{Im}\lambda_1}, \quad C = \text{const}.$$

(ii) If $\operatorname{Im}\lambda_{N_0} = \dfrac{1}{2}$, we choose $\epsilon > 0$ such that the straight line $\operatorname{Im}\lambda_{N_0} = \dfrac{1}{2} + \epsilon$ does not contain eigenvalues of the spectral problem (4.4), (4.5) and $-\dfrac{1}{2} < \operatorname{Im}\lambda_1 < \ldots$ $\ldots < \operatorname{Im}\lambda_{N_0} < \dfrac{1}{2} + \epsilon$. Repeating arguments analogous to the proof of Proposition 3.3, we obtain

$$u(x,t) = \sum_{j=1}^{N_0} c_j(t) r^{\operatorname{Im}\lambda_j} \phi_j(\omega) + u_0(x,t),$$

where ϕ_j are infinitely differetiable functions of ω, and $c_j \in L_{2,\mathrm{loc}}[0,\infty)$, $j = 1, \ldots, N_0$, and $u_0 \in H^{2,0}_0(e^{-\gamma_1 t}, \Omega_\infty)$.

By using arguments analogous to the proof of case (i), we obtain

$$|u(x,t)| \leqslant C r^{\operatorname{Im}\lambda_1}, \quad C = \text{const}.$$

If Ω is convex in a neighbourhood of the the coordinate origin, then the strip $-\dfrac{1}{2} \leqslant \operatorname{Im}\lambda < 1$ does not contain eigenvalues of the spectral problem (4.4), (4.5) (see [3, p. 290]). It follows from Theorem 2.2 that $u(x,t) \in H^2_0(e^{-\gamma_2 t}, \Omega_\infty)$.

From above arguments we have the following proposition.

Proposition 4.2. *Let u(x,t) be a generalized solution of the problem* (4.1) – (4.3) *in the space* $\overset{\circ}{H}{}^{1,0}(e^{-\gamma t}, \Omega_\infty)$, *and let* $f, f_t, f_{tt}, f_{ttt} \in L^\infty(0,\infty; L_2(\Omega))$, $f(x,0) = f_t(x,0) = f_{tt}(x,0) = 0$. *Then*

$$|u(x,t)| \leqslant C|x|^{\operatorname{Im}\lambda_1}, \quad C = \text{const}.$$

Moreover, if Ω *is convex in a neighbourhood of the the coordinate origin then* $u(x,t) \in H^2_0(e^{-\gamma_2 t}, \Omega_\infty)$.

4.3. Case $n > 3$. In this case, the strip

$$1 - \frac{n}{2} \leqslant \operatorname{Im}\lambda \leqslant 2 - \frac{n}{2}$$

does not contain eigenvalues of the spectral problem (4.4), (4.5) (see [3, p. 289]).

From Theorem 2.2, we obtain the following proposition.

Proposition 4.3. *Let* $u(x,t)$ *be a generalized solution of the problem* (4.1) – (4.3) *in the space* $\overset{\circ}{H}{}^{1,0}(e^{-\gamma t}, \Omega_\infty)$, *and let* $f, f_t, f_{tt}, f_{ttt} \in L^\infty(0,\infty; L_2(\Omega))$, $f(x,0) = f_t(x,0) = f_{tt}(x,0) = 0$. *Then* $u \in H^2_0(e^{-\gamma_2 t}, \Omega_\infty)$.

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